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PROPAGATION OF ELECTROMAGNETIC WAVES FOR ARBITRARY DEPENDENCE OF MAGNETIC PERMEABILITY ON MAGNETIC INDUCTION*

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The paper deals with simple plane-polarized waves in magnetic media. It is assumed that the magnetic permeability of the medium is a given function of the modulus of its magnetic induction /1/. The qualitative form of this dependence makes it possible to determine the magnitude of magnetic field intensity under which the wave becomes inverted. The discontinuities in such media are studied and conditions under which they become evolutionary are found. A plane self-similar boundary value problem is solved. Similar waves and discontinuities in the electromagnetic field parameters in magnetic media with magnetic permeability linearly and inversely proportional to the modulus of magnetic field intensity, have been investigated in /2/.

1. Simple waves. Let us consider the propagation of an electromagnetic wave through an infinite magnetic medium which magnetizes under the action of an external magnetic field of strength $\mathbf{H}$, according to the law $\mathbf{B} = \mu(H)\mathbf{H}$, in the case when the magnetic permeability μ is a function of the modulus of magnetic field intensity. The permittivity ε of the medium is assumed constant and equal to unity.

The propagation of plane electromagnetic waves in a magnetic medium is described by the Maxwell equations(it is assumed that all quantities depend only on time t and the coordinate x along which the wave propagates)

$$\frac{\partial H_y}{\partial x} - \frac{1}{c}\frac{\partial E_z}{\partial t} = 0, \quad \frac{\partial E_z}{\partial x} - \frac{1}{c}\frac{\partial B_y}{\partial t} = 0, \quad \frac{\partial H_z}{\partial x} + \frac{1}{c}\frac{\partial E_y}{\partial t} = 0, \quad \frac{\partial E_y}{\partial x} + \frac{1}{c}\frac{\partial B_z}{\partial t} = 0, \quad \frac{\partial B_x}{\partial x} = 0, \quad \frac{\partial E_x}{\partial x} = 0 \quad (1.1)$$

From the last two equations of the syatem (1.1) follows $B_x = B_{x0}, E_x = E_{x0}$. If a dependence $\mu = \mu(H)$ exists, then the equation $\mathbf{B} = \mu\mathbf{H}$ yields the relation $\mathbf{H} = \mathbf{H}(\mathbf{B})$.

Two types of electromagnetic waves can be obtained from the system (1.1): 1) transverse waves propagating with velocity $a_A = c\sqrt{H/B}$ (such waves were studied in detail in /2/); 2) plane-polarized waves in which the magnetic permeability of the medium varies. In what follows, we shall only consider the plane-polarized waves.

We choose the coordinate system so that $H_z \equiv 0$. If follows that $B_z \equiv 0$ and in this case we have $E_y =$ const. Let Fig.1 be a qualitative representation of the dependence of H on B. We shall assume that $dH/dB > 0$ everywhere and that only a single inflection point B_* exists at which $d^2H/dB^2 = 0$. Moreover $d^2H/dB^2 < 0$ when $0 < B < B_*$ and $d^2H/dB^2 > 0$ when $B > B_*$.

We denote by B_{**} the value of B for which the relation $d\mu^{-1}/dB = H'/B - H/B^2 = 0$ holds. Such a dependence of H on B is characteristic for the process of magnetizing a completely demagnetized medium /1/.

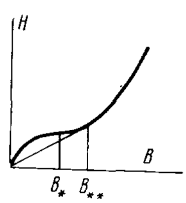

Fig.1

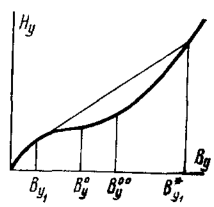

Fig.2

When $B_x = B_{x0}$ and $B_z = 0$, the dependence of H_y on B_y can be written in the form

$$H_y = H_y(B_y) = H(B)B_y/B, \quad B = \sqrt{B_y^2 + B_{x0}^2}$$

We have

$$dH_y/dB_y = F(B_y) = (dH/dB)B_y^2/B^2 + HB_{x0}^2/B^3 > 0$$

From the system (1.1) we can obtain the value of the rate of propagation a of a longitudinal electromagnetic wave representing the charactersitic velocity of the system

$$F\frac{\partial B_y}{\partial x} - \frac{1}{c}\frac{\partial E_z}{\partial t} = 0, \quad \frac{\partial E_z}{\partial x} - \frac{1}{c}\frac{\partial B_y}{\partial t} = 0 \quad (1.2)$$

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The characteristic velocity is given, in accordance with (1.2), by the formula $a = \pm c\sqrt{F}$, and the equality $a = a_A$ holds at the point $B = B_{**}$ since at this point $d\mu^{-1} / dB = 0$ and hence $dH / dB = H / B$. The system (1.2) can be used to obtain the relation connecting the electric and magnetic fields in a simple longitudinal wave

$$E_z = \mp \int \sqrt{F}\, dB_y$$

The inversion of simple waves is governed by the sign of the derivative dF / dB_y

$$\frac{dF}{dB_y} = \frac{d^2}{dB_y^2}\left(\frac{HB_y}{B}\right) = \sin\theta\left(\frac{d^2H}{dB^2}\sin^2\theta + 3\frac{d\mu^{-1}}{dB}\cos^2\theta\right), \qquad \sin\theta = B_y / B, \quad \cos\theta = B_{x0} / B \tag{1.3}$$

We assume that the magnetization curve (Fig.1) is such, that d^2H / dB^2 and $d\mu^{-1} / dB$ change their sign once when $B > 0$, i.e. $d^2H / dB^2 |_{B=B_*} = 0$, $d\mu^{-1} / dB |_{B=B_{**}} = 0$, retain the negative sign when $B < B_*$ and $B < B_{**}$ respectively, and the positive sign when $B > B_*$ and $B > B_{**}$. Therefore from (1.3) it follows that the function dF / dB_y vanishes on the segment $[B_* B_{**}]$.

Let us find the conditions under which the form of the curve $B_y(H_y)$ will coincide qualitatively with the curve $B(H)$, i.e. under which $dF / dB_y = 0$ only at one point of the segment $[B_* B_{**}]$. We shall denote this point by B_y°. The condition that $d^2F / dB_y^2 > 0$ at the point at which $dF / dB_y = 0$, represents the sufficiency. We have

$$\frac{d^2F}{dB_y} = \sin^2\theta\left(H''' \sin^2\theta + 4\frac{H''}{B} + \cos^2\theta \frac{H''}{B}\right) \tag{1.4}$$

In the case when the function H'' is monotonous, i.e., $H''' > 0$, the right hand part of (1.4) is always positive. If H''' changes its sign, then we must demand that the expression within the brackets in (1.4) be positive on the segment $[B_* B_{**}]$. In this case the relation $H_y(B_y)$ has the same qualitative form as the curve $H(B)$ (see Fig.1). Let us denote by B_y^∞ the value of B_y for which $d\mu^{-1} / dB_y = 0$. We have

$$\frac{d\mu^{-1}}{dB_y} = \frac{dH/B}{dB_y} = \frac{1}{B}\frac{dH}{dB}\frac{dB}{dB_y} - \frac{H}{B^2}\frac{dB}{dB_y} = \frac{1}{B}\left(H' - \frac{H}{B}\right)\sin\theta = \frac{d\mu^{-1}}{dB}\sin\theta$$

Consequently, if $d\mu^{-1}/dB = 0$ at the point $B = B_{**}$, then $d\mu^{-1} / dB_y = 0$ at the same point B_{**} and we obviously have $B_y = B_{**}\sin\theta$. Thus when $B_y < B_y^\circ$ ($B_y > B_y^\circ$), the waves which invert are those in which the magnetic field behind the wave decreases (increases).

2. Shock waves. Conditions at the strong discontinuities in the values of the electromagnetic field following from the Maxwell equations, have the following form in the case when surface currents and charges are absent from the discontinuities /3/:

$$[B_x] = 0, \; [E_x] = 0, \qquad [\mathbf{H}_\tau] = c^{-1}(\mathbf{v} \times \mathbf{E}), \quad [\mathbf{E}_\tau] = c^{-1}(\mathbf{B} \times \mathbf{v}) \tag{2.1}$$

The x-axis is chosen normal to the discontinuity, and $\mathbf{v} = v\mathbf{n}$ is the normal component of the velocity of propagation of the discontinuity through the magnetic medium. Square brackets denote the differences in the values of the quantities behind (denoted below by the index 2) and in front (denoted by index 1) of the discontinuity. System (2.1) yields a plane-polarized discontinuity moving with velocity $v = c\sqrt{[H_y] / [B_y]}$ in which the components of the electromagnetic field H_y, B_y and E_z vary, and a transverse Alfven discontinuity moving with velocity $v_A = c\sqrt{H / B}$ in which rotation of the vectors $\mathbf{H}, \mathbf{B}$ and $\mathbf{E}$ takes place without affecting their moduli. Such a discontinuity is discussed in /2/. In what follows, we shall limit ourselves to such plane discontinuities. The square of the propagation velocity of such a discontinuity is proportional to the tangent of the angle of inclination of the secant connecting, on the curve $H_y = H_y(B_y)$, the points corresponding to the states in front of, and behind the discontinuity.

The rate of propagation of the simple waves $a = c\sqrt{dH_y / dB_y}$ is shown graphically by the angle of inclination of the tangent at the point $B_{y1} H_{y1}$ to the curve $H_y = H_y(B_y)$. Therefore the condition of evolutionarity with respect to the plane-polarized perturbations can be fulfilled, if the secant corresponding to the velocity of the shock wave lies on one side of this curve, otherwise the rate of propagation of the discontinuity will be less than the velocity of propagation of the plane-polarized perturbations in front of, as well as behind the discontinuity.

For a secant lying above (below) the curve $H_y = H_y(B_y)$, the shock waves with $B_{y2} > B_{y1}$ (with $B_{y2} < B_{y1}$) are evolutionary.

When $B_{y1} < B_y^\circ$, all discontinuities with $0 \leqslant B_{y2} \leqslant B_{y1}$ and $B_{y2} > B_{y1}^*$ are evolutionary. The point B_{y1}^* (Fig.2) is found from the Jouguet condition

$$v(B_{y1}^*) = a(B_{y1}) \tag{2.2}$$

when $B_y^\circ < B_{y1} < B_y^\infty$, all discontinuities with $B_{y2} > B_{y1}$ and $B_{y2} < B_{y1}^*$ are evolutionary. The Jouguet condition (2.2) holds at the discontinuity with $B_{y2} = B_{y1}^*$ from above. When $B_{y1} \geqslant B_y^\infty$, all discontinuities with $B_{y2} > B_{y1}$ are evolutionary.

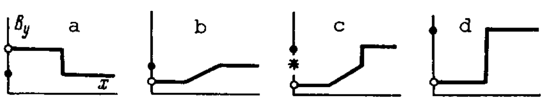

Fig.3

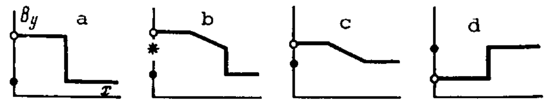

Fig.4

All discontinuities are evolutionary with respect to transverse perturbations, since the transverse waves move with velocity $a_A = c\sqrt{H / B}$ which is represented graphically by the angle of inclination of the straight line segment connecting the coordinate origin with the point on the curve $H_y = H_y(B_y)$ corresponding to the state behind, or in front of the discontinuity. The necessary condition for the discontinuity to be evolutionary with respect to the transverse waves is, that the secant, the inclination of which oorresponds to the velocity of propagation of the shock wave, lies on one side of the segments connecting the points of inersection of the curve by the secant, with the coordinate origin. Fig.2 shows that this condition always holds.

3. Plane self-similar boundary value problem. Let B_{y1} be the magnitude of the magnetic field in the magnetic medium at $x = 0$, and B_{y2} denote the magnitude of the magnetic field in the region $x > 0$. We fix the value of B_{y1} and vary B_{y2}, and investigate the self-similar solutions of the boundary value problem, remembering that a shock wave, a straight wave, or their combination, can all pass through the magnetic medium.

1°. Let $0 < B_{y1} < B_y^\circ$, and let us consider all possible values of B_{y2}. (The results are shown in Fig.3 where the light points, dark points and asterisks denote, respectively, the ordinates B_{y1}, B_{y2} and B_{y2}^*).

a) $0 < B_{y2} < B_{y1}$. In this case we have a discontinuity of the electromagnetic field in which the field jumps from the value B_{y1} to the value B_{y2}.

b) $B_{y1} < B_{y2} < B_{y1}^\circ$. The field parameters vary continuously in a simple wave.

c) $B_y^\circ < B_{y2} < B_{y1}^*$. B_{y1}^* is determined from the condition (2.2). A discontinuity will exist up to the value B_{y2}^*, and after this a simple wave will propagate through the magnetic medium; here we have $a(B_{y2}^*) = v(B_{y2})$.

d) $B_{y2} > B_{y1}^*$. In this case a discontinuity is formed across which the value of the field changes from B_{y1} to B_{y2}.

2°. Let $B_y^\circ < B_{y1} < B_y^{\circ\circ}$. We vary the field B_{y2} as a parameter, and the results are shown in Fig.4 where the notation of Fig.3 is used.

a) $B_{y2} < B_{y1}^*$. B_{y1}^* is found from the condition (2.2). A discontinuity across which the field will jump from B_{y1} to B_{y2} will propagate through the magnetic medium.

b) $B_{y1}^* < B_{y2} < B_y^\circ$. In this case the field will vary up to the value B_{y2}^* determined by the condition $a(B_{y2}) = v(B_{y2})$ and the jump will move from the point B_{y2}^* to B_{y2}.

c) $B_y^\circ < B_{y2} < B_{y1}$. The field parameters vary continuously in a simple wave.

d) $B_{y2} > B_{y1}$. A shock wave will exist in which the parameters vary in a discontinuous manner.

The self-similar boundary value problem for $B_{y1} > B_y^{\circ\circ}$ is solved as in Sect.2, except that in this case the solution 2°, a) will be absent since no point B_{y1}^* exists which satisfies the condition (2.2).

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